Exploiting Geometry in the SINR_k Model*

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Abstract

We introduce the SINR_k model, which is a practical version of the SINR (signal to interference plus noise ratio) model. In the SINR_k model, in order to determine whether s's signal is received at c, where s is a sender and c is a receiver, one only considers the k most significant senders w.r.t. to c (other than s). Assuming uniform power, these are the k closest senders to c (other than s). Under this model, we consider the well-studied scheduling problem: Given a set L of sender-receiver requests, find a partition of L into a minimum number of subsets (rounds), such that in each subset all requests can be satisfied simultaneously. We present an $O(1)$-approximation algorithm for the scheduling problem (under the SINR_k model). For comparison, the best known approximation ratio under the SINR model is $O(\log n)$. We also present an $O(1)$-approximation algorithm for the maximum capacity problem (i.e., for the single round problem), obtaining a constant of approximation which is considerably better than those obtained under the SINR model. Finally, for the special case where $k = 1$, we present a PTAS (polynomial-time approximation scheme) for the maximum capacity problem. Our algorithms are based on geometric analysis of the SINR_k model. Lemmas and proofs will be omitted due to limited space. They can be found in the full paper that was accepted to and presented at ALGOSENSORS 2014 [1]. The entire list of references can also be found in the full paper.

1 Introduction and notation

Let $L = \{(c_1, s_1), (c_2, s_2), \ldots, (c_n, s_n)\}$ be a set of n pairs of points in the plane representing n (directional) links, where the points $c_1, \ldots, c_n$ represent the receivers and the points $s_1, \ldots, s_n$ represent the senders. The *length* of the link $(c_i, s_i) \in L$ is the Euclidean distance between c_i and s_i and is denoted l_i. We denote the Euclidean distance between c_i and s_j, for $j \neq i$, by l_{ij}. The set of all receivers is denoted $C = C(L)$ and the set of all senders is denoted $S = S(L)$. Let p_i be the transmission power of sender s_i. Finally, let S_i^k be the set of the k most significant (closest) senders w.r.t. to c_i (other than s_i). In the SINR_k model, a link (c_i, s_i) is *feasible*, if c_i receives the signal sent by s_i. That is, if the following inequality holds:

$$\frac{p_i / l_i^\alpha}{\sum\limits_{\{j : s_j \in S_i^k\}} p_j / l_{ij}^\alpha + N} \geq \beta$$

where $\alpha, \beta \geq 1$ and $N \geq 0$ are constants (α is the path-loss exponent, N is the ambient noise, and β is the threshold above which a signal is received successfully). We will make common assumptions that (i) $p_i = p_j$, for $1 \leq i, j \leq n$, i.e., uniform power (see, e.g., [2,3]), and (ii) $N = 0$, i.e., there is no ambient noise (see, e.g., [4]).

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The *scheduling problem* is to partition the set of links L into a minimum number of *feasible* subsets (i.e., rounds), where a subset L_i is *feasible* if, when only the senders in $S(L_i)$ are active, each of the links in L_i is feasible. A greedy algorithm that successively finds a feasible subset of maximum cardinality of the yet unscheduled links yields an $O(\log n)$-approximation. Therefore, it is interesting to first focus on the *maximum capacity* problem, i.e., find a feasible subset of L of maximum cardinality. In other words, find a set $Q \subseteq L$, such that if only the senders in $S(Q)$ are active, then each of the links in Q is feasible, and Q is of maximum cardinality.

2 Maximum capacity problem

Algorithm 1 An $O(1)$-approximation

```
𝒬 ← ∅
Sort 𝒟 by the radii of the disks in increasing order.
while 𝒟 ≠ ∅ do
   Let D be the smallest disk in 𝒟
   𝒟 ← 𝒟 \ {D}
   for all D′ ∈ 𝒟, such that D ∩ D′ ≠ ∅ do
      𝒟 ← 𝒟 \ {D′}
   𝒬 ← 𝒬 ∪ {D}
return 𝒬
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Theorem 1 ([1]). *Given a set L of n links and a constant k, one can compute a $(1/8\tau)$-approximation for the maximum capacity problem under the SINR$_k$ model, where* $\tau = \frac{2\pi(k+1)}{\arctan(\frac{\sqrt[\alpha]{\beta k}-1}{\sqrt[\alpha]{\beta k}+1})}$

Note that in the maximum capacity problem where any sender and receiver can be paired, Algorithm 1 also gives a $(1/8\tau)$-approximation.

3 Scheduling

The following is due to a result by Miller et al. [5] showing how to color an intersection graph of a set of balls in $\mathbb{R}^d$ of bounded ply.

Theorem 2 ([1]). *Given a set L of n links and a constant k, one can compute a $(9\tau + 1)$-approximation for the scheduling problem under the SINR$_k$ model.*

4 A PTAS for maximum capacity with $k = 1$

The following can be achieved due to a technique discovered by Timothy Chan [6]

Theorem 3 ([1]). *Given a set L of n links and $\varepsilon > 0$, one can compute a $(1 - \varepsilon)$-approximation for the maximum capacity problem under the SINR$_1$ model.*

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